French Clock Beat Rates

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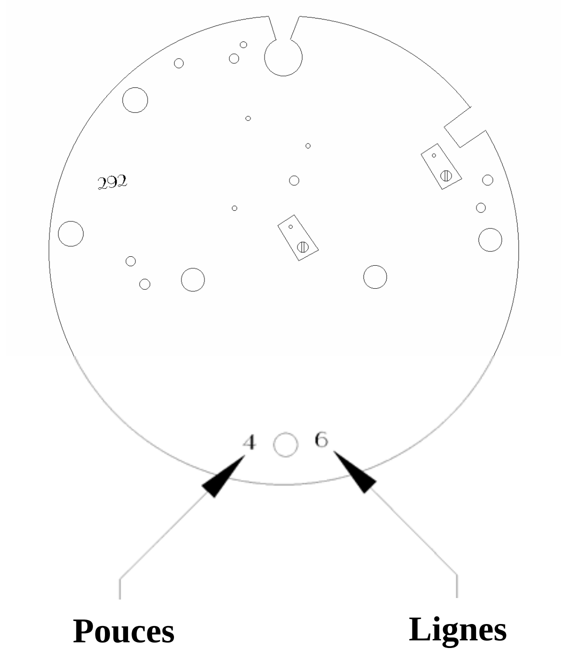
With the advent of the new digital timing machines it has become easier to rate a clock using its beat rate. Beat rate is defined as the number of blows of the escape wheel teeth, as they fall on the lock faces of the pallets, in a given time period. This time period is usually measured in hours or minutes. The beat rate in most clock timers is in beats per hour or BPH. In order for a clock to keep time, the pendulum length must be matched to the gear ratios of the movement to achieve a correct beat rate. The beat rate for a specific clock to keep time is a constant no matter where you are in the world. Using a timer to make adjustments to the pendulum length to get the correct beat rate is a fast, easy method and can be very accurate.

Beat rate clock timers have one flaw however, namely that the beat rate must be known for each and every clock being timed. The timer I use comes with beat rate tables for a variety of clocks. There is also a method for determining beat rate based on train calculations if the specific clock isn't listed. Although the calculations are fairly straight forward, I don't usually want to take the time to count the teeth and pinion leaves in a gear train for every clock not listed in the tables. If the beat rate isn't readily available, I'll often rate it the old-fashioned way; by letting it run and making adjustments. I've even tried changing the beat rate an even increment, say 100 BPH, and noting the change in rate. This too takes time. With this in mind, I have tried to make life easier by constructing a list of beat rates for French clocks based on the numbers for pendulum lengths found on the back plates of a majority of French movements. (see fig. 1)



(fig. 1) Typical French Movement Back Plate
showing the location of the pendulum length numbers
corresponding to the “pouces” and “lignes”.

Several items must be taken into account when compiling a table of this nature. First is the problem of determining the values of the numbers originally used in calculating the pendulum lengths. The French used "pouce", the French word meaning "inch", and "ligne", meaning "line", as units of measurement for their pendulum lengths with a pouce being divided into 12 lignes. Therefore, if one ligne is added to a pendulum length of 4 pouces,11 lignes the result is 5 pouces, not 4,12. These units have an integral part in the history of France's conversion to the metric system and some of the difficulty in finding the original values is due to this conversion. If the values the French used to determine pendulum length are not used in determining the beat rates, the rates will be inaccurate and defeat the purpose of the table. It is important, therefore, to use the late 1800's pouce and ligne values in determining the beat rate for a late 1800's clock.

Prior to the French Revolution (1789 - 1799), the pouce and ligne were accepted units of measurement with the value of a pouce in millimeters being 27.07 mm and a ligne, 2.256 mm. However, during this time, the scientific community was trying to establish a new decimal or "metric" system to replace the cumbersome one using pouce and ligne. The political unrest at the time gave them an uphill battle, and in 1800 a decree was made to use the new metric system with the old French names to help it gain acceptance. This turned out to be unpopular and for twelve years two "metric" systems were in use: one with pouce and ligne to represent metric units and one with units of meter and centimeter. Then in 1812, Napoleon established the "systeme usuelle" which modified the metric weights and measures to bring them more in line with the pre-Revolution units. This changed the value of a pouce from 27.07 mm to 27.75 mm and a ligne from 2.256 mm to 2.31 mm. Finally, in 1840, the "usuelle" system was repealed in favor of the scientist's first metric system and France declared the decimal metric system to be the only legal one. Anyone who ignored this law was subject to stiff penalties.[1] This system is the same one used today in many countries of the world and the scientific community as a whole. France is the center of the metric or International System (SI) of weights and measures and maintains the standards for the world. So, why did French clockmakers stamp the numbers for pouces and lignes instead of centimeters and millimeters on the back plates of their movements after 1840? Perhaps it was a little left-over rebellion, stubbornness, or more likely, tradition.

Variations in acceleration due to gravity (*g*) also play a role in determining the length of a pendulum, and it is important to understand how that relates to beat rate. The variation in *g* is greatest when traveling in a North-South direction rather than in an East-West. 65% of the variation when traveling North-South is due to the rotational affect of the earth. This will have the effect of counteracting *g*. 35% is due to the Equatorial Bulge which changes your distance from the center of the earth[2] (see table 1). Other affects on *g* are changes in altitude (see table 2), and local variations due to the Earth's non-uniform surface density. As you move about the earth, *g* increases or decreases. This means that a pendulum will have to be lengthened or shortened in order to obtain a correct beat rate. It is for this reason that we must consider what the French used as the value for the acceleration due to gravity (*g*). Was it the value of *g* in London, Paris, the equator, the poles, or somewhere else? We must assume that the numbers stamped on the back plate represent a specific measurable length, and that the length was calculated using known equations which rely on the value of *g*.

Table 1--The different values of g over various latitudes.[3]

Latitude	*g* (circa 1916)	Pendulum Length (PL)
0 deg	385.034 "/s^2	39.0121"
10 deg	385.099 "/s^2	39.0184"
20 deg	385.274 "/s^2	39.0365"
30 deg	385.548 "/s^2	39.0642"
40 deg	385.884 "/s^2	39.0982"
50 deg	386.240 "/s^2	39.1344"
60 deg	386.576 "/s^2	39.1683"
70 deg	386.850 "/s^2	39.1960"
80 deg	387.028 "/s^2	39.2141"
90 deg (poles)	387.090 "/s^2	39.2204"

Table 2--The different values of g over various altitudes.

Altitude at 45 deg. latitude	*g* (circa 1982)	(PL)
0.000 miles	386.063 "/s^2	39.1164"
0.621 miles	385.945 "/s^2	39.1044"
2.486 miles	385.591 "/s^2	39.0685"
4.971 miles	385.118 "/s^2	39.0206"
9.942 miles	384.134 "/s^2	38.9209"
19.884 miles	382.283 "/s^2	38.7334"
62.137 miles	377.953 "/s^2	38.2946"
310.686 miles	335.827 "/s^2	34.0264"
(altitude of a satellite)		
621.321 miles	291.732 "/s^2	29.5587"

So, what does the value of *g* have to do with determining beat rate?

The value for acceleration due to gravity (*g*) is derived from Newton's second law of motion:

F = ma

where: *F* = force, *m* = mass, and *a* = acceleration

Let acceleration (a) equal acceleration due to gravity (*g*). *a* = *g*.

Then F = ma becomes:

F = mg

Now consider Newton's law of gravitation:

$$\mathbf{F = G \frac{m_1 m_2}{r^2}}$$

Where: *F* = force between two particles having mass, m_1 = mass of particle 1, m_2 = mass of particle 2, *r* = distance between the two particles, and *G* = universal gravitational constant = 6.6726 x 10^{-11} m^3/kg s^2 (the above value for this constant was adopted in 1982 and first accurately measured by Lord Cavendish in 1798)

Letting m_1 be the mass of our pendulum (m), m_2 be the mass of the earth (M_e), and r the radius of the earth (R_e), we get:

$$\mathbf{F = G \frac{mM_e}{R_e^2} = m \frac{GM_e}{R_e^2}}$$

Therefore, from Newton's second law of motion F = mg, we see from the above equation:

$$\mathbf{g = \frac{GM_e}{R_e^2}}$$ which is not a constant and varies with location. (see table 1 & 2)

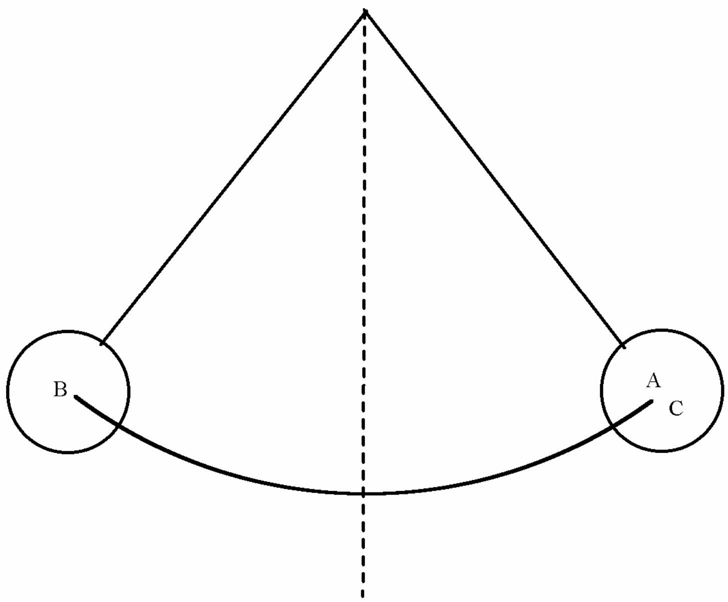
Where:

$\mathbf{G = 6.6726 \times 10^{-11}\ m^3/kg\ s^2}$
$\mathbf{M_e = 5.97 \cdot 10^{24}\ kg\ \ or\ 6.6 \cdot 10^{21}\ tons}$
$\mathbf{R_e = 6.37 \cdot 10^{6}\ m\ \ or\ 2.09 \cdot 10^{7}\ ft}$
(1982 values)

In physics, the period of oscillation of a simple pendulum (T) is equal to two times pi (π), times the square root of the pendulum length (PL), divided by the acceleration due to gravity (g).

$$\mathbf{T = 2\pi \sqrt{PL/g}}$$

Here, "period of oscillation" of a simple pendulum is taken as the time for the *total* angular travel of the pendulum. (see fig. 2)



(fig. 2) Period of Oscillation vs. Beat
One period of oscillation is the time it takes the pendulum to travel the arc ABC. One beat is equal to half the period of oscillation or the time it takes the pendulum to travel the arc AB or BC.

This would mean that <u>one beat</u> is <u>half</u> of the period of oscillation since an escape wheel tooth receives a blow twice in each period of oscillation. To make matters more confusing, this half-period is also widely used as T. For clarity, I will use $T_{1/2}$. Then we have:

$$\mathbf{T_{1/2} = \pi \sqrt{PL/g}}$$

If we let $T_{1/2}$ = 1 sec. per beat and g = *local* acceleration due to gravity, the length of a seconds pendulum can be determined at our location. (see table 3)

$$\mathbf{1\ sec/beat = \pi \sqrt{PL/g}}$$

Inverting we get beats per second (BPS):

$$\mathbf{1\ BPS = \frac{\sqrt{g}}{\pi \sqrt{PL}} \quad or \quad 1\ BPM = \frac{60 \cdot \sqrt{g}}{\pi \sqrt{PL}}}$$

Squaring both sides gives:

$$\mathbf{1\ BPM^2 = \frac{3600 \cdot g}{\pi^2 \cdot PL}}$$

Now, let the pendulum constant $P_C = \dfrac{3600 \cdot g}{\pi^2}$

Then:

$$\mathbf{1\ BPM^2 = \frac{P_C}{PL} \quad or \quad PL = \frac{P_C}{BPM^2}}$$

Which is dependent on location because P_C is dependent on g.

Converting to BPH:

$$\mathbf{1\ BPH^2 = \frac{3600 \cdot P_C}{PL}}$$

And finally:

$$\mathbf{1\ BPH = \sqrt{(3600 \cdot P_C/PL)}}$$

Table 3--PL values for a seconds pendulum.[4]

	PL (")	P_C	g ("/sec^2)	g (m/sec^2)
Equator	39	140400	384.9146	9.7768
New York	39.1012	140764	385.9125	9.8022
Paris	39.13	140868	386.1976	9.8094
London	39.14	140904	386.2963	9.8119
Greenland	39.20	141120	386.8885	9.8270
Poles	39.206	141142	386.9488	9.8285

We can derive a beat rate from the equation based on pendulum length and local pendulum constant, provided the numbers used for PL and P_C are accurate.

Table 4, which is a list of beat rates based on the numbers found on the rear plates of a majority of French movements, was compiled with this equation. I have endeavored to use historical values to keep the beat rates as accurate as possible. The numbers are as follows:

1 pouce = 27.06996 mm (late 1800's value[1])
1 ligne = 2.25583 mm (late 1800's value[5])
P_C = 140904 (1905 value for London[4])

I chose the pendulum constant (P_C) for London because the beat rates generated using this constant come closer to the actual rates than the P_C for Paris. It is highly probable that the French manufacturers used the P_C for London since England was setting the standards at the turn of the century. Even today, the world over, we use standards set in England; eg. the use of Greenwich Mean Time, Standard Time, and Universal Time.

The pendulum length for these calculations was found by multiplying the left of the two digits on the back plate by the "pouce factor" and adding that to the result found when the right digit was multiplied by the "ligne factor". This result is in millimeters and must be converted to inches before computing the beat rate. It should be assumed that a single number refers to the pouce on a plate having only one number for pendulum length and that the lignes would equal zero.

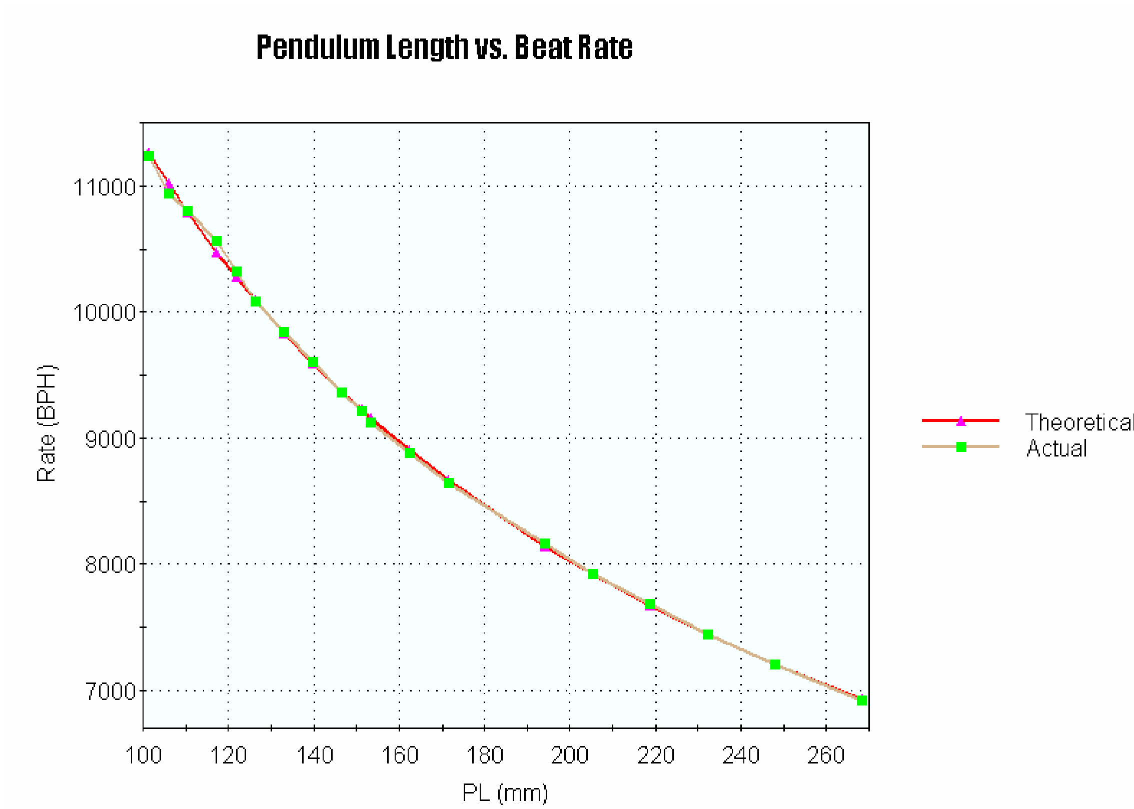


Fig 3. Graph showing how close the theoretical values of BPH match gear train calculated (actual) values.[6]

The equation is as follows:

$$PL'' = \frac{N_P \cdot 27.06996\ \text{mm} + N_L \cdot 2.25583\ \text{mm}}{25.4\ \text{mm}/''}$$

Where: PL" = pendulum length in inches
N_P = the number of pouces.
N_L = the number of lignes.
(see fig. 1)

Several French clock train counts have been used as models in determining the accuracy of the beat rate table. Using the above equation to calculate BPH will generate a curve that gradually diverges from the actual beat rate function and a closer result will be found using an equation that has been corrected for the drift. (see fig. 3.)

The final, corrected equation is as follows:

$$1\ \text{BPH} = 0.994078 * \sqrt{3600 \cdot P_C} * \left(\frac{N_P \cdot 27.06996\ \text{mm} + N_L \cdot 2.25583\ \text{mm}}{25.4\ \text{mm}/''} \right)^{-0.5}$$

This equation will produce a result which proves to be with in 1% of a gear train calculated result. A 1% error in BPH is roughly a 15 minute per day error in rate. Remember, the purpose of this table is to aid in rating a French clock. If the clock doesn't rate close to the suggested BPH then there may be other factors involved such as altered, damaged, or switched parts. What counts is that the minute hand goes around the dial once every 60 minutes for the duration of its run.

Table 4--French Clock Beat Rates Based on Plate Numbers

Pouces	Lignes	BPH	PL (mm)	PL (“)	$T_{1/2}$ (sec/beat)
3	0	12588	81.21	3.197	0.285979
3	1	12417	83.47	3.286	0.289924
3	2	12253	85.72	3.375	0.293816
3	3	12094	87.98	3.464	0.297657
3	4	11942	90.23	3.552	0.301449
3	5	11796	92.49	3.641	0.305193
3	6	11655	94.74	3.730	0.308893
3	7	11518	97.00	3.819	0.312548
3	8	11387	99.26	3.908	0.316162
3	9	11259	101.51	3.997	0.319734
3	10	11136	103.77	4.085	0.323268
3	11	11017	106.02	4.174	0.326762
4	0	10902	108.28	4.263	0.330220
4	1	10790	110.54	4.352	0.333642
4	2	10682	112.79	4.441	0.337030
4	3	10576	115.05	4.529	0.340383
4	4	10474	117.30	4.618	0.343704
4	5	10375	119.56	4.707	0.346993
4	6	10278	121.81	4.796	0.350252
4	7	10184	124.07	4.885	0.353480
4	8	10093	126.33	4.973	0.356679
4	9	10004	128.58	5.062	0.359849
4	10	9918	130.84	5.151	0.362992
4	11	9833	133.09	5.240	0.366108
5	0	9751	135.35	5.329	0.369198
5	1	9671	137.61	5.418	0.372261
5	2	9592	139.86	5.506	0.375300
5	3	9516	142.12	5.595	0.378315
5	4	9441	144.37	5.684	0.381306
5	5	9368	146.63	5.773	0.384273
5	6	9297	148.88	5.862	0.387218
5	7	9227	151.14	5.950	0.390140
5	8	9159	153.40	6.039	0.393041
5	9	9093	155.65	6.128	0.395920
5	10	9028	157.91	6.217	0.398779
5	11	8964	160.16	6.306	0.401617
6	0	8901	162.42	6.394	0.404436
6	1	8840	164.68	6.483	0.407235
6	2	8780	166.93	6.572	0.410014
6	3	8721	169.19	6.661	0.412775
6	4	8664	171.44	6.750	0.415518
6	5	8607	173.70	6.839	0.418243
6	6	8552	175.95	6.927	0.420950

Pouces	Lignes	BPH	PL (mm)	PL (“)	$T_{1/2}$ (sec/beat)
6	7	8498	178.21	7.016	0.423640
6	8	8445	180.47	7.105	0.426313
6	9	8392	182.72	7.194	0.428969
6	10	8341	184.98	7.283	0.431609
6	11	8290	187.23	7.371	0.434232
7	0	8241	189.49	7.460	0.436840
7	1	8192	191.75	7.549	0.439433
7	2	8145	194.00	7.638	0.442010
7	3	8098	196.26	7.727	0.444573
7	4	8052	198.51	7.815	0.447120
7	5	8006	200.77	7.904	0.449654
7	6	7962	203.02	7.993	0.452173
7	7	7918	205.28	8.082	0.454678
7	8	7875	207.54	8.171	0.457169
7	9	7832	209.79	8.260	0.459647
7	10	7790	212.05	8.348	0.462112
7	11	7749	214.30	8.437	0.464563
8	0	7709	216.56	8.526	0.467002
8	1	7669	218.82	8.615	0.469428
8	2	7630	221.07	8.704	0.471842
8	3	7591	223.33	8.792	0.474243
8	4	7553	225.58	8.881	0.476632
8	5	7516	227.84	8.970	0.479009
8	6	7479	230.09	9.059	0.481375
8	7	7442	232.35	9.148	0.483729
8	8	7406	234.61	9.236	0.486071
8	9	7371	236.86	9.325	0.488402
8	10	7336	239.12	9.414	0.490723
8	11	7302	241.37	9.503	0.493032
9	0	7268	243.63	9.592	0.495330
9	1	7234	245.89	9.681	0.497618
9	2	7202	248.14	9.769	0.499896
9	3	7169	250.40	9.858	0.502163
9	4	7137	252.65	9.947	0.504420
9	5	7105	254.91	10.036	0.506667
9	6	7074	257.16	10.125	0.508904
9	7	7043	259.42	10.213	0.511131
9	8	7013	261.68	10.302	0.513348
9	9	6983	263.93	10.391	0.515556
9	10	6953	266.19	10.480	0.517755
9	11	6924	268.44	10.569	0.519944
10	0	6895	270.70	10.657	0.522124

Ref. 1. *Revolution in Measurement: Western European Weights and Measures Since the Age of Science*; Zupko, Ronald Edward; The American Philosophical Society, Independence Square, Philadelphia; 1990; pp. 169 - 175.
2. *Fundamentals of Physics* Second ed.; Halliday and Resnick; John Wiley & Sons, New York; 1981; pp. 218 - 254.
3. *Handbook of Chemistry and Physics, 5th ed.*; The Chemical Rubber Company; Cleveland, Ohio; 1916; pp. 244 - 249.
4. *The Modern Clock*; Goodrich; North American Watch Tool & Supply Co., IL; 1905; p. 12.
5. *Treatise on Modern Horology*; Saunier; Charles T. Branford Co.; Vewton Centre, MA; 1878; p. 819
6. Generated using the National Center for Educational Statisitics' "Create a Graph" web site: http://nces.ed.gov/nceskids/graphing/